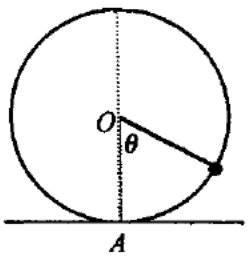
Take $g = 9{\cdot}8$ ms^{-2} and give all answers correct to 3 significant figures where necessary.

1. A particle of mass m kg is attached to one end of a light inextensible string of length l m whose other end is fixed to a point O. The particle is made to move in a vertical circle with centre O, with **constant** angular velocity ω rad s^{-1}. At a certain instant it is in the position shown, where the string makes an angle θ radians with the downward vertical through O.



(a) Find an expression, in terms of m, l and ω, for the kinetic energy of the particle at this instant. **(2 marks)**

(b) Find an expression, in terms of m, g, l and θ, for the potential energy of the particle relative to the horizontal plane through the lowest point A. **(2 marks)**

(c) Determine the position of the particle when the rate of increase of its total energy, with respect to time, is a maximum. **(3 marks)**

2. A particle moves along a straight line in such a way that its displacement x m from a fixed point O on the line, at time t seconds after it leaves O, is given by $x = p \sin \omega t + q \cos \omega t$ where p, q and ω are constants.

(a) Show that the motion of the particle is simple harmonic. **(5 marks)**

(b) If the particle leaves O with speed 15 ms^{-1}, and $\omega = 3$, find the amplitude of the motion. **(2 marks)**

3. A particle P of mass 0·2 kg moves in a horizontal circle on one end of an elastic string whose other end is attached to a fixed point O. The angular velocity of P is π rad s^{-1}. The natural length of the string is 1 m and, while P is in motion, the distance $OP = 1{\cdot}15$ m.

(a) Calculate, to 3 significant figures, the modulus of elasticity of the string. **(6 marks)**

The motion now ceases and P hangs at rest vertically below O.

(b) Show that the extension in the string in this position is about 13 cm. **(3 marks)**

4. A small stone P of mass m kg is attached to one end of a light elastic string of modulus $3mg$ N and natural length $2l$ m. The other end of the string is fixed to a point O at a height $3l$ m above a horizontal surface. P is released from rest at O; it hits the surface and rebounds to a height of $2l$ m. The coefficient of restitution between P and the surface is e.

Calculate the value of e. **(9 marks)**

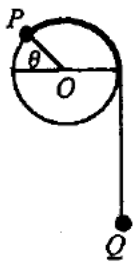
State one assumption (other than the string being light) that you have used in your solution. **(1 mark)**

5. A small sphere S, of mass m kg is released from rest at the surface of a liquid in a right circular cylinder whose axis is vertical. When S is moving downwards with speed v ms^{-1}, the viscous resistive force acting upwards on it has magnitude v^2 N.

   (a) Write down a differential equation for the motion of S, clearly defining any symbol(s) that you introduce. **(4 marks)**

   (b) Find, in terms of m, the distance S has fallen when its speed is $\sqrt{\frac{mg}{2}}$ ms^{-1}. **(9 marks)**

6. The diagram shows two identical particles, each of mass m kg, connected by a thin, light inextensible string. P slides on the surface of a smooth right circular cylinder fixed with its axis, through O, horizontal. Q moves vertically. OP makes an angle θ radians with the horizontal.

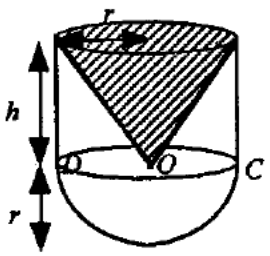


   The system is released from rest in the position where $\theta = 0$.

   (a) Show that the vertical distance moved by Q is $\frac{\theta}{\sin\theta}$ times the vertical distance moved by P. **(4 marks)**

   (b) In the position where $\theta = \frac{\pi}{6}$, prove that the reaction of the cylinder on P has magnitude $\left(1-\frac{\pi}{6}\right)mg$ N. **(9 marks)**

7. A container consists of two sections made from the same material : a hollow portion formed by removing a cone (shaded in the figure) from a solid cylinder of radius r and height h, and a solid hemisphere of radius r. The vertex of the removed cone coincides with the centre O of the horizontal plane face of the hemisphere. CD is a diameter of this plane face.



   (a) Show that the distance of the centre of mass of the container from the plane face of the hemisphere is $\left|\frac{3}{8}(h-r)\right|$. Explain why the modulus sign is necessary. **(9 marks)**

   (b) Find the ratio $h : r$ in each of the following cases :

   (i) When the container is suspended from the point C, the angle made by CD with the vertical is equal to the angle which CD would make with the vertical if the hemisphere *alone* were suspended from C. **(4 marks)**

   (ii) The container is able to stand without toppling in any position when it is placed with the surface of the hemispherical part in contact with a smooth horizontal table. **(3 marks)**

